

QCD Sum Rule Calculation of Twist-4 Corrections to Bjorken and Ellis-Jaffe Sum Rules

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Abstract: We calculate the twist-4 corrections to the integral of $g_1(x, Q^2)$ in the framework of QCD sum rules using an interpolating nucleon field which contains explicitly a gluonic degree of freedom. This information can be used together with previous calculations of the twist-3 contribution to the second moment of $g_2(x)$ to estimate the higher-twist corrections to the Ellis-Jaffe and Bjorken sum rules. We get $f^{(2)}(proton) = -0.037 \pm 0.006$ and $f^{(2)}(neutron) = -0.013 \pm 0.006$. Numerically our results roughly agree with those obtained by Balitsky, Braun and Kolesnichenko based on a sum rule for a simpler current. Our calculations are far more stable as tested within the sum rule approach but are more sensitive to less well known condensates.

Spin effects in strong interaction high energy processes are one of the best tools to study QCD both in the perturbative and non perturbative regime. It has become clear that the reliable determination of the Q^2 dependence, both due to radiative corrections and due to higher twist contributions is a central task of QCD theory. The Q^2 dependence of spin variables is in general more benign than for unpolarized quantities (e. g. the anomalous dimension for the Bjorken sum rule vanishes), allowing to extract very interesting information from data taken at Q^2 as low as 1 GeV². While the EMC [1] and SMC [2] experiments have still comparatively large Q^2 , SLAC [3] data are taken down at a rather low mean Q^2 of about 2 GeV². The proton data indicated a disagreement with the polarized-proton sum rule, the Ellis-Jaffe sum rule [4], leading to a lot of excitement in the high-energy physics community. Since experiments now not only provide data for the spin-dependent structure function of the proton $g_1^p(x)$ but also of the neutron $g_1^n(x)$ one of the most solid predictions of QCD, the Bjorken sum rule [5], is tested experimentally. However, for low Q^2 this sum rule (strictly valid in the asymptotic Bjorken limit) receives corrections. The most familiar are those from perturbative QCD, calculated for the leading twist term up to order α_S^3 [6] and estimated to order α_S^4 with an rough estimate of order α_S^5 [7]. Higher-twist corrections are given in terms of complicated hadronic matrix-elements and are suppressed by powers of Q^2 . Since experiments still are not in a save region of asymptotically large Q^2 all this corrections have to be examined carefully to give a complete picture of the Q^2 dependence of the Bjorken sum rule [8].

The first moment of $g_1(x, Q^2)$ at fixed Q^2 is given by [9]

$$\int_0^1 dx g_1(x, Q^2) = \frac{1}{2}a^{(0)} + \frac{m_N^2}{9Q^2} \left(a^{(2)} + 4d^{(2)} + 4f^{(2)} \right) + O\left(\frac{m_N^4}{Q^4}\right). \quad (1)$$

In the above formula we have not included higher order corrections to the coefficient functions which can be written as power series in $\alpha_S(Q^2)$.¹ In the present paper we focus on the correction proportional to $f^{(2)}$ which is defined by the matrix element of the twist-4 operator

$$O_\sigma(0) = \bar{q}(0)g\tilde{G}_{\sigma\beta}(0)\gamma^\beta q(0) \quad (2)$$

¹The radiative corrections to the leading-twist singlet part were calculated to order α_S^2 [10] and estimated to order α_S^3 [11].

in the nucleon state $|pS\rangle$ of momentum p and spin S , $p^2 = -S^2 = m_N^2$, $p \cdot S = 0$,

$$\langle pS|O_\sigma(0)|pS\rangle = 2m_N^2 f^{(2)} S_\sigma. \quad (3)$$

Here and in the following we temporarily neglect the normalization-point dependence of the operator O_σ .

The reduced matrix elements $d^{(2)}$ and $f^{(2)}$ can be expressed through the second moments of the polarized nucleon structure functions $g_1(x)$ and $g_2(x)$ [12]:

$$\begin{aligned} \int dx x^2 g_2(x) &= \frac{1}{3} (d^{(2)} - a^{(2)}) \\ \int dx x^2 g_1(x) &= \frac{1}{2} a^{(2)} \end{aligned} \quad (4)$$

While $a^{(2)}$ may be taken directly from experiments, the matrix elements $f^{(2)}$ and $d^{(2)}$ were first estimated by Balitsky, Braun and Kolesnichenko [13] using the QCD sum rules technique. In our previous work [14] we presented an independent sum rule calculation of $d^{(2)}$ which essentially confirmed the values obtained by BBK. The calculation of $f^{(2)}$ presented in this paper completes therefore the program of estimating the leading power corrections to the sum rules for $\int g_1(x, Q^2) dx$.

Before we are going to dwell on the details of the calculation we would like to stress that the leading higher-twist matrix elements describe fundamental properties of the nucleon. The twist-4 operator eq. (2) is a measure for the contribution of the collective gluonic field to the spin of the nucleon. Writing the dual field strength tensor in its components we get

$$\langle pS| -B_A^\sigma j_A^0 + (\vec{j}_A \times \vec{E}_A)^\sigma |pS\rangle = 2m_N^2 f^{(2)} S^\sigma \quad (5)$$

where the quark-current is denoted as $j_A^\mu = -g\bar{q}\gamma^\mu t^A q$ and B_A^σ and E_A^σ are the colour magnetic and colour electric fields. In the rest system of the nucleon an analogous relation holds for the twist-3 operator which determines $d^{(2)}$

$$\langle pS| 2B_A^\sigma j_A^0 + (\vec{j}_A \times \vec{E}_A)^\sigma |pS\rangle = 8m_N^2 d^{(2)} S^\sigma. \quad (6)$$

Knowledge of $d^{(2)}$ and $f^{(2)}$ then allows to estimate magnetic and electric field contributions to the spin separately.

In the usual approach nucleon matrix elements of local operators can be extracted from a three-point correlation function

$$\Pi_{\Gamma}(p) = i^2 \int d^4x e^{ipx} \int d^4y \langle 0 | T \{ \eta(x) O_{\Gamma}(y) \bar{\eta}(0) \} | 0 \rangle \quad (7)$$

which involves an interpolating current $\eta(x)$ with a certain overlap λ between the state created from the vacuum by $\eta(x)$ and the nucleon state

$$\langle 0 | \eta(x) | pS \rangle = \lambda u(p, S) \exp(ipx). \quad (8)$$

The overlap integral can be determined from an additional two-point correlation function

$$\Pi(p) = i \int d^4x e^{ipx} \langle 0 | T \{ \eta(x) \bar{\eta}(0) \} | 0 \rangle. \quad (9)$$

In practical application it is often advantageous to consider the ratio of three- and two-point correlation functions such that the λ -dependence cancels out.

For QCD sum rule calculations of nucleon properties the standard choice for $\eta(x)$ has been for a long time the three-quark current introduced by Ioffe [15]

$$\eta_I(x) = \left[u^a(x) C \gamma_{\mu} u^b(x) \right] \gamma_5 \gamma^{\mu} d^c(x) \varepsilon^{abc}, \quad (10)$$

which was used in the calculation of $f^{(2)}$ and $d^{(2)}$ by Balitsky, Braun and Kolesnichenko (BBK) [13]. As explained in [16, 14] for the investigation of operators which, like O_{σ} eq. (2), contain explicitly gluonic degrees of freedom it is very useful to match these by an interpolating nucleon current that contains gluonic degrees of freedom as well. For such a proton current we chose

$$\eta_G(x) = \frac{2}{3} \left(\eta_G^{\text{old}}(x) - \eta_G^{\text{ex}}(x) \right), \quad (11)$$

where

$$\eta_G^{\text{old}}(x) = \varepsilon^{abc} \left(u^a(x) C \gamma_{\mu} u^b(x) \right) \gamma_5 \gamma^{\mu} \sigma_{\alpha\beta} \left[G^{\alpha\beta}(x) d(x) \right]^c, \quad (12)$$

and

$$\eta_G^{\text{ex}}(x) = \varepsilon^{abc} \left(u^a(x) C \gamma_{\mu} d^b(x) \right) \gamma_5 \gamma^{\mu} \sigma_{\alpha\beta} \left[G^{\alpha\beta}(x) u(x) \right]^c. \quad (13)$$

This current was first studied in [16] and tested in the calculation of the nucleon gluonic form factor and the total momentum fraction carried by gluons. Next the twist-3 correction $d^{(2)}$ occurring in the expansion of $\int g_1(x) dx$ was

predicted [14]. Note that the current (11) may be regarded as the leading expansion term of a non-local version of the classical three-quark current (10). Working with non-local sources has become also popular in lattice-gauge theories where stability can be increased by taking a number of derivatives of the quark-fields.

Using lattice-gauge theory the three- and two-point correlators (7) and (9) can be calculated directly from first principles. Such a project is pursued by the Jülich-group [17]. The basic idea of the QCD sum rule technique on the other hand is to employ the duality between the hadronic and partonic representation of a correlation function and to extract the quantity of interest by demanding that both descriptions match each other at some intermediate scale. If $-p^2$ is sufficiently large the main contribution comes from small x -distances of order of $x^2 \sim 1/(-p^2)$. The consideration of contributions from different t -channel distances is more involved. When $y^2 \leq 1/(-p^2)$ the standard machinery of the short-distance expansion is applicable resulting in the known expansion in terms of quark and gluon condensates. However, the contribution from large y^2 has to be accounted for separately.

The solution to this problem was first formulated by Balitsky [18]. The Operator Product Expansion (OPE) of a three-point correlation function has a twofold structure. Terms of the first type come from the region $x^2 \sim y^2 \sim 1/(-p^2)$ and are proportional to vacuum expectation values (VEV) of local gauge-invariant operators multiplied by coefficient functions depending on p^2 . In the following we shall refer to these terms as to local power corrections (LPC).

Terms of the second type called bilocal power corrections (BPC) originate from distances $y^2 \gg x^2 \sim 1/(-p^2)$. To treat such contributions properly one should expand the time-ordered product of nucleon interpolating currents

$$T(\eta_I(x)\bar{\eta}_G(0)) = \sum_n C_n^{BL}(x)\tilde{O}_n(0), \quad (14)$$

in a series of local, gauge-invariant operators $\tilde{O}_n(0)$ of increasing dimension n . When this expansion is inserted back in (7) it results, together with standard LPC, in the following general form of the OPE of the three-point correlator (7)

$$\Pi_\Gamma(p) = \sum_n c_{\Gamma,n}^L(p) \langle O_n^L \rangle + \sum_n c_n^{BL}(p) \Pi_{\Gamma,n}^{BL}(0). \quad (15)$$

The bilocal power corrections are determined by the long-distance, non-perturbative contributions to correlation functions at zero momentum [19]:

$$\Pi_{\Gamma,n}^{\text{BL}}(q) = i \int d^4y e^{iqy} \langle 0 | T \{ O_{\Gamma}(y) \tilde{O}_n(0) \} | 0 \rangle, \quad (16)$$

with $q = 0$. Special care has to be taken to properly eliminate the perturbative short-distance singularities from these correlators which are already included in LPC coefficients $c_{\Gamma,n}^L(p)$. We stress again that the essence of the expansion (15) is the separation of different scales: vacuum expectation values of local operators $\langle O^L \rangle$ and correlators Π^{BL} describe long distance effects while the coefficients c^L and c^{BL} receive contribution only from highly virtual quark and gluon fields which propagate for small distances. The OPE of the correlation function Π_{Γ} must be written as the sum of both LPC's and BPC's, as above. We note that in general only this sum has a physical meaning and is independent of the regularisation scheme. Contrary to the case of our previous calculation of the twist-3 correction [14], the BPC's do play a crucial role in the analysis of the twist-4 matrix element.

The right hand side of (7) can be decomposed into different Lorentz structures according to

$$\begin{aligned} & i \int d^4x e^{ipx} i \int d^4y \langle 0 | T \{ \eta_I(x) \bar{q}(y) g \tilde{G}_{\sigma\mu}(y) \gamma^{\mu} q(y) \bar{\eta}_G(0) \} | 0 \rangle \\ & = p_{\sigma} \not{p} \gamma_5 W^{\mathcal{A}}(p^2) + \gamma_{\sigma} \gamma_5 W^{\mathcal{B}}(p^2) + \dots, \end{aligned} \quad (17)$$

where the ellipses stand for other terms which can be eliminated taking the trace of (17) with an appropriate projector. The invariant functions W^i , $i = \mathcal{A}, \mathcal{B}$ can be represented as spectral integrals [19]

$$\begin{aligned} W^i(p^2) &= \int ds \frac{1}{s - p^2} \left[a^i \delta'(s - m_N^2) + b^i \delta(s - m_N^2) + \Theta(s - s_0) \rho^c(s) \right] \\ &+ \text{subtractions}. \end{aligned} \quad (18)$$

We have accepted the conventional “resonance plus continuum” model of the spectral density with the continuum density $\rho^c(s)$ dual to all graphs with non-vanishing imaginary part at $-p^2 \rightarrow \infty$. The constant a^i which stands in front of the double-pole term is proportional to $f^{(2)}$ while the single-pole term is determined by nucleon-to-continuum transitions and has to be eliminated from the final answer. In principle the information about the magnitude of

$f^{(2)}$ can be extracted either from $W^{\mathcal{A}}$ or from $W^{\mathcal{B}}$. In practical calculations it is advantageous to consider $W^{\mathcal{A}}$ because of its lower dimensionality and this structure was chosen for the analysis presented in [13]. We realize that the QCD sum rule approach involves a number of approximations the accuracy of which is sometimes difficult to assess a priori. To get a better feeling of the intrinsic uncertainties of the whole method we have decided to analyze $W^{\mathcal{B}}$ which leads to a sum rule which is more sensitive to the higher mass region in the spectral representation (18). The constant coefficient in front of the double-pole nucleon contribution to $W^{\mathcal{B}}$ can be found to be equal to

$$a^{\mathcal{B}} = -2f^{(2)}\lambda_I\lambda_G m_N^6. \quad (19)$$

Here, λ_I and λ_G are the overlap of the Ioffe current (10) respectively the quark - gluon current (11) with the nucleon, while m_N denotes the nucleon mass.

The invariant function $W^{\mathcal{B}}$ can be easily projected out of the three-point correlator (7) by taking the trace with $\frac{1}{4}\gamma_5 \not{S}$ and choosing momentum p_μ such that $p \cdot S = 0$. The expansion of $W^{\mathcal{B}}$ in LPC is standard and we are not going to dwell on the details here. The net result can be written as the following formula:

$$\begin{aligned} 4W_{\text{LPC}}^{\mathcal{B}}(p^2) = & \\ & A \frac{\alpha_S}{\pi^5} (-p^2)^4 \log(-p^2/\mu^2) + B \frac{\alpha_S}{\pi} \langle \bar{q}q \rangle^2 (-p^2) \log(-p^2/\mu^2) \\ & + C \frac{1}{\pi^4} \langle f g^3 GGG \rangle (-p^2) \log(-p^2/\mu^2) \\ & + F \frac{\alpha_S}{\pi} m_0^2 \langle \bar{q}q \rangle^2 \\ & + G \frac{\langle \bar{q}q \rangle^2 \langle g^2 GG \rangle}{-p^2} + H \frac{\langle \bar{q}q \rangle^2 m_0^4}{-p^2} \end{aligned} \quad (20)$$

where the numerical coefficients are given in Table 1. Note that the coefficient in front of the gluon condensate $\langle GG \rangle$ vanishes.

The calculation of BPC's is more involved and will be described next in some details. Let us consider operators which may contribute two-point correlators (16) of dim-6 and dim-10 to the sum rule. The expansion of $T(\eta_I(x)\bar{\eta}_G(0))$ in local operators leads to the following series:

	A	B	C	D	G	H
d	-1/2592	-4/3	0	88/81	5/81	1/9
u	-1/3240	-20/27	-23/2304	1411/2592	1/81	-1/54

Table 1: Numerical coefficients corresponding to LPC contributing to the sum rule eq. 20 . The upper line gives the values for the twist-4 operator involving d quarks, the lower line for the corresponding operator with u-quarks

$$\begin{aligned}
T(\eta_I(x)\bar{\eta}_G(0)) &= \frac{20x_\sigma}{3\pi^4x^8}\gamma_5 \not{x} \left[i \left(\bar{d}\gamma_5\gamma_\sigma g G^{\sigma\sigma} d \right) (0) - \left(\bar{d}\gamma_\sigma g \tilde{G}^{\sigma\sigma} d \right) (0) \right] \\
&+ \frac{16x_\sigma}{3\pi^4x^8}\gamma_5 \not{x} \left[i \left(\bar{u}\gamma_5\gamma_\sigma g G^{\sigma\sigma} u \right) (0) - \left(\bar{u}\gamma_\sigma g \tilde{G}^{\sigma\sigma} u \right) (0) \right] \\
&+ m_0^2 < \bar{q}q > \frac{x_\alpha x_\lambda}{6\pi^2x^4}\gamma_\varphi\gamma_5 \left[2 \left(\bar{u}\gamma_5\sigma^{\varphi\lambda} D^\alpha u \right) (0) - \frac{1}{3} \left(\bar{d}\gamma_5\sigma^{\varphi\lambda} D^\alpha d \right) (0) \right. \\
&- \left. i g^{\varphi\lambda} \left(\bar{d}\gamma_5 D^\alpha d \right) (0) \right] \\
&- \alpha_S < \bar{q}q >^2 \frac{8}{27\pi x^4} \left(5x^\varphi x_\alpha - 9g_\alpha^\varphi x^2 \right) \gamma_\varphi\gamma_5 \left(\bar{u}\gamma^\alpha\gamma_5 u \right) (0) \\
&- \alpha_S < \bar{q}q >^2 \frac{32x^\varphi x_\alpha}{27\pi x^4} \gamma_\varphi\gamma_5 \left(\bar{u}\gamma^\alpha\gamma_5 u \right) (0) \\
&- < \bar{q}q > \frac{4i}{9\pi^2} \frac{x_\lambda x_\alpha}{x^4} \gamma_\varphi\gamma_5 \left[\left(\bar{u}g\tilde{G}^{\lambda\varphi} D^\alpha u \right) (0) + 2 \left(\bar{d}g\tilde{G}^{\lambda\varphi} D^\alpha d \right) (0) \right] + \dots
\end{aligned} \tag{21}$$

The ellipses on the right hand side denote gamma-structures other than $\gamma_\varphi\gamma_5$ that do not contribute to W^B . Furthermore in the above formula we did not write explicitly the operator $< \bar{q}q > (q\sigma^{\mu\nu}g\tilde{G}^{\sigma\sigma}D^\alpha q)(0)$ due to the complicated and lengthy structure of its coefficient. In the end its contribution to the final sum rule turns out to be small.

The relevant BPC arise from the correlation function of the local operators present on the right-hand side of eq. (21) with the twist-4 operator which defines $f^{(2)}$ at zero momentum. In the particular case of operators which are proportional to equations of motion of QCD, like the operators $\bar{q}\gamma_5\gamma^\sigma G_{\sigma\sigma}q$, $\bar{q}\gamma_5\sigma_{\varphi\lambda}D_\alpha q$ and $\bar{q}\gamma_5 D_\alpha q$, the correlation functions can be evaluated by means of exact low-energy theorems. Using the functional integral

representation it is easy to obtain the following Ward identities, see e. g. [20]:

$$\begin{aligned}
i \int d^4 y \langle 0 | T O_\sigma(y) (\bar{q} \gamma_5 D_\alpha q) (0) | 0 \rangle &= 0 \\
i \int d^4 y \langle 0 | T O_\sigma(y) (\bar{q} \gamma_5 \sigma_{\varphi\lambda} D_\alpha q) (0) | 0 \rangle &= m_0^2 \langle \bar{q} q \rangle \frac{1}{12} (g_{\sigma\lambda} g_{\alpha\phi} - g_{\sigma\phi} g_{\alpha\lambda}) \\
i \int d^4 y \langle 0 | T O_\sigma(y) (\bar{q} \gamma_5 \gamma^\rho g G_{\rho\alpha} q) (0) | 0 \rangle &= \frac{4\pi}{3} \alpha_S \langle \bar{q} q \rangle^2 g_{\sigma\alpha}
\end{aligned} \tag{22}$$

To avoid misunderstanding we note that the above identities should be understood in the following way. The large-distance contribution to the lhs is equal to the large distance contribution to the rhs which is just determined by a non-perturbative VEV of the corresponding operator.

The other remaining two-point correlation functions at zero-momentum transfer cannot be evaluated exactly. Instead, they may be estimated by considering additional two-point sum rules. Since ultimately one is interested in their long-distance behaviour, we focus on the contribution coming from the lowest possible intermediate state - the massless, chiral, pseudoscalar meson. One can define the overlap of the operators given in (21) with a pion state as:

$$\begin{aligned}
\langle 0 | \bar{q} \gamma_\mu \gamma_5 q | \pi(p) \rangle &= i f_\pi p_\mu \\
\langle 0 | \bar{q} g \tilde{G}_{\mu\nu} \gamma^\nu q | \pi(p) \rangle &= i f_\pi \delta_\pi^2 p_\mu \\
\langle 0 | \bar{q} g \tilde{G}_{\mu\nu} D^\nu q | \pi(p) \rangle &= i f_\pi \tilde{\delta}_\pi^3 p_\mu \\
\langle 0 | \bar{q} \sigma_{\mu\rho} g \tilde{G}^{\rho\sigma} D_\sigma q | \pi(p) \rangle &= i f_\pi \bar{\delta}_\pi^3 p_\mu
\end{aligned} \tag{23}$$

The constants δ_π , $\tilde{\delta}_\pi$ and $\bar{\delta}_\pi$ can be calculated by evaluating two-point correlation functions in the sum rule framework at *non-zero* momentum. We stress that to avoid double counting special care has to be taken to subtract properly contributions of higher excited states like the A_1 -meson or the continuum.

Note that consideration of the contribution arising from pion exchange alone is sufficient as long as we consider only the flavor non-singlet combination $f^{(2)}(NS) = f^{(2)}(u) - f^{(2)}(d)$. Consideration of the flavor singlet combination $f^{(2)}(S) = f^{(2)}(u) + f^{(2)}(d)$ naturally forces us to take into account contributions from η and η' . In the chiral limit of massless quarks which we employ in this paper the η can be considered as massless. The axial anomaly, however, will give rise to the η' mass [21]. The pedestrian

solution to this problem would be to disregard the η' contribution as a short-distance one but the real physics is certainly more complicated [23]. As our calculation offers no insight into this complicated problem we have decided simply to include the η and the η' on the same footing as the pion, but it should be kept in mind that due to the axial anomaly the prediction for $f^{(2)}(S)$ is subject to an uncertainty which is probably small, but presently unresolvable.

As far as δ_π is concerned an estimate due to Novikov et. al. [22] can be found in the literature. We essentially repeated their calculation for correlators which determine $\tilde{\delta}_\pi$ and $\bar{\delta}_\pi$ with the result $\delta_\pi^2 = 0.21 \text{ GeV}^2$, $\tilde{\delta}_\pi^3 = 0.033 \text{ GeV}^3$, $\bar{\delta}_\pi^3 = -0.1 \text{ GeV}^3$, $f_\pi = 133 \text{ MeV}$. Hence, the contribution of a massless chiral boson to the correlation functions at zero momentum can be estimated as:

$$\begin{aligned} i \int d^4y \langle 0 | T O_\sigma(y) (\bar{q} \gamma_\alpha \gamma_5 q) (0) | 0 \rangle &= \frac{3}{4} f_\pi^2 \delta^2 g_{\alpha\sigma} \\ i \int d^4y \langle 0 | T O_\sigma(y) i (\bar{q} g \tilde{G}_{\alpha\mu} D^\mu q) (0) | 0 \rangle &= -f_\pi^2 \delta_\pi^2 \tilde{\delta}_\pi^3 g_{\alpha\sigma} \\ i \int d^4y \langle 0 | T O_\sigma(y) (\bar{q} \sigma_{\alpha\varrho} g \tilde{G}^{\varrho\mu} D_\mu q) (0) | 0 \rangle &= -f_\pi^2 \delta_\pi^2 \bar{\delta}_\pi^3 g_{\alpha\sigma} \end{aligned} \quad (24)$$

The factor $\frac{3}{4}$ in the correlator of O_σ with the axial current is due to current conservation, see the discussion above. The remaining correlation function $\Pi = i \int d^4y \langle 0 | T O_\sigma(y) O^\sigma(0) | 0 \rangle$ was evaluated with the help of an additional sum rule, see [13], to be $\Pi \sim 3 \cdot 10^{-3} \text{ GeV}^6$. After Fourier transforming eq. (21) and inserting it back into eq. (15) we finally obtain the expansion of $W^B(p^2)$ in terms of both local and bilocal power corrections:

$$\begin{aligned} 4W^B(p^2) &= \\ A \frac{\alpha_S}{\pi^5} (-p^2)^4 \log(-p^2/\mu^2) + (B + B_{BL}) \frac{\alpha_S}{\pi} <\bar{q}q>^2 (-p^2) \log(-p^2/\mu^2) \\ &+ C \frac{1}{\pi^4} <fg^3 GGG> (-p^2) \log(-p^2/\mu^2) \\ &+ D \Pi \frac{1}{\pi^2} (-p^2) \log(-p^2/\mu^2) + F \frac{\alpha_S}{\pi} m_0^2 <\bar{q}q>^2 \\ &+ G \frac{<\bar{q}q>^2 <g^2 G^2>}{-p^2} + (H + H_{BL}) \frac{<\bar{q}q>^2 m_0^4}{-p^2} \\ &+ I \pi \alpha_S <\bar{q}q>^2 f_\pi^2 \delta^2 \frac{1}{-p^2} + (J \tilde{\delta}_\pi^3 + K \bar{\delta}_\pi^3) <\bar{q}q> f_\pi^2 \delta^2 \frac{1}{-p^2} \end{aligned} \quad (25)$$

	B_{BL}	D	H_{BL}	I	J	K
d	-20/ 27	5/36	-1/27	-64/9	$2^6/27$	5/9
u	-16/ 27	1/9	2/9	208/9	$2^5/27$	7/36

Table 2: Numerical coefficients corresponding to BPC contributing to the sum rule eq. 25 . The upper line gives the values for the twist-4 operator involving d quarks, the lower line for the corresponding operator with u-quarks

The numerical coefficients corresponding to the BPC can be read off from table 2.

Before we proceed to extract the matrix element $f^{(2)}$ one comment has to be made. An inspection of eq. (25) reveals an important difference with respect to the previous calculation of [13]. The theoretical side of the present sum rule given by eq. (25) is manifestly free from effects of mixing of the three-point correlation function (7) with two-point correlators [13, 20]. In other words the use of the quark-gluon current (11) resulted in a much milder singularity structure of the three-point correlator than in the case with the three-quark current. The mixing, which occurs already at the tree-level i.e., before genuine radiative corrections are considered, produces extra UV logarithms in Wilson coefficients and makes it necessary to use a more complicated model of the spectral representation [24], although the resulting corrections turn out to be much smaller than the overall uncertainties. In the present case the representation (18) is sufficient to adequately reproduce all terms arising from the theoretical calculation.

To extract the matrix element of interest we employ the standard strategy of QCD sum rules. First we multiply the sum rule by $m_N^2 - p^2$ to eliminate the single-pole term in (18) and then apply a Borel transformation to both sides, arriving at the following expression

$$\begin{aligned}
& -8f^{(2)}\lambda_I\lambda_G m_N^6 e^{-m_N^2/M^2} \\
& = \tilde{A}(5! E_6 M^{12} - 4! E_5 M^{10} m_N^2) + \tilde{B}(M^4 m_N^2 E_2 - 2M^6 E_3) + \\
& \tilde{C}(M^4 E_2 - M^2 m_N^2 E_1) + \tilde{D}m_N^2
\end{aligned} \tag{26}$$

where $\tilde{A}, \tilde{B}, \tilde{C}$ and \tilde{D} are the coefficients in front of $(-p^2)^4 \log(-p^2/\mu^2)$, $(-p^2) \log(-p^2/\mu^2)$, $\log(-p^2/\mu^2)$ and $1/(-p^2)$ respectively. $E_n(s_0, M^2)$ de-

notes as usual

$$E_n = 1 - e^{(-s_0/M^2)} \sum_{k=0}^{n-1} \frac{1}{k!} \left(s_0/M^2\right)^k. \quad (27)$$

The dependence on the overlap integrals λ_I and λ_G can be eliminated from eq. (26) by dividing the three-point sum rule by the two-point sum rule derived in [16]

$$\begin{aligned} 2(2\pi)^4 m_N^2 \lambda_I \lambda_G e^{-m_N^2/M^2} &= \frac{6}{5} \frac{\alpha_S}{\pi} M^8 E_4 \\ + \frac{1}{2} < g^2 GG > M^4 E_2 - \frac{4}{3} \frac{\alpha_s}{\pi} (2\pi)^4 < \bar{q}q >^2 M^2 E_1 + \frac{2}{3} (2\pi)^4 m_0^2 < \bar{q}q >^2 \end{aligned} \quad (28)$$

The used set of condensates $< \bar{q}q > = (-0.257 \text{ GeV})^3$, $< \alpha_s/\pi GG > = 0.012 \text{ GeV}^4$, $m_0^2 = < \bar{q}g\sigma Gq > / < \bar{q}q > = 0.65 \text{ GeV}^2$ and $< fg^3 GGG > = 0.046 \text{ GeV}^6$ correspond to the standard ITEP values rescaled to the normalization point $\mu_0^2 \sim m_N^2 \sim 1 \text{ GeV}^2$. The strong coupling constant at 1 GeV is taken to be $\alpha_S = 0.37$ ($\Lambda = 150$) MeV. The continuum threshold is chosen as $s_0 = (1.5 \text{ GeV})^2$ roughly corresponding to the Roper resonance position.

The quotient sum rule for the matrix element of the twist-4 operator has been plotted in Fig. 1. This figure shows the singlet (S) and nonsinglet (NS) part of $f^{(2)}$. $f^{(2)}(S) = f^{(2)}(u) + f^{(2)}(d)$, $f^{(2)}(NS) = f^{(2)}(u) - f^{(2)}(d)$. For comparison the corresponding sum rules obtained from the analysis in [13] which employed Ioffe currents only are also shown. The square of the overlap integral λ_I^2 is determined from the additional two-point sum rule

$$\begin{aligned} 2(2\pi)^4 \lambda_I^2 e^{-m_N^2/M^2} &= M^6 E_3 \\ + \frac{1}{4} < g^2 GG > M^2 E_1 + \frac{4}{3} (2\pi)^4 < \bar{q}q >^2 \end{aligned} \quad (29)$$

which is just the standard sum rule considered by Ioffe. Instead of using a fixed value for λ_I^2 we divided the sum rule obtained in [13] by the sum rule (29). The final values of the matrix elements can be estimated from the figures by taking $M^2 \approx m_N^2 \approx 1 \text{ GeV}^2 \approx \mu_0^2$, where μ_0^2 represents the normalization point of the matrix element (2). Numerically, their values are:

$$\begin{aligned} f^{(2)}(S) &= -0.09 \pm 0.02 & f^{(2)}(NS) &= -0.07 \pm 0.02 \\ f^{(2)}(proton) &= -0.037 \pm 0.006 \\ f^{(2)}(neutron) &= -0.013 \pm 0.006 \end{aligned} \quad (30)$$

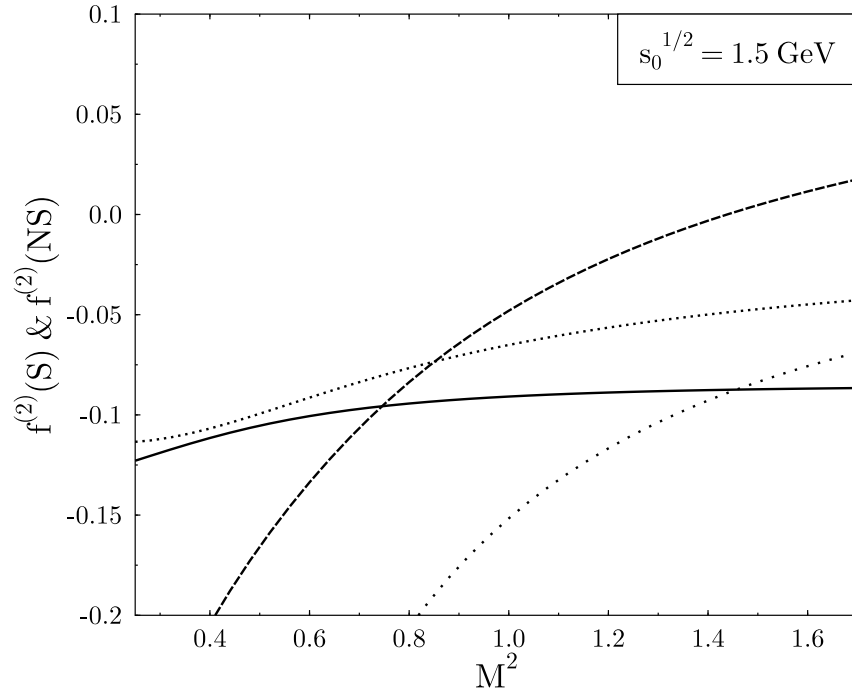


Figure 1: Stability plot of the sum rule eq. (26). The full line corresponds to $f^{(2)}(S)$, the dotted line to $f^{(2)}(NS)$. For comparison the results of the analysis in [13] are also shown. The dashed line corresponds to $f_{BBK}^{(2)}(S)$ the space-dotted line to $f_{BBK}^{(2)}(NS)$.

at $\mu_0^2 \approx 1 \text{ GeV}^2$. These values are to be compared with those obtained from the sum rules given in [13]:

$$\begin{aligned} f_{\text{BBK}}^{(2)}(S) &= -0.068 \pm 0.03 & f_{\text{BBK}}^{(2)}(NS) &= -0.18 \pm 0.04 \\ f_{\text{BBK}}^{(2)}(\textit{proton}) &= -0.049 \pm 0.01 \\ f_{\text{BBK}}^{(2)}(\textit{neutron}) &= 0.01 \pm 0.01 \end{aligned}$$

All errors given are only due to the dependence on the Borel parameter M^2 . An additional error enters due to the factorization of high dimensional condensates. For condensates of dimension 8 the generally accepted error is estimated to be of order $\sim 20\%$. There is unfortunately very little experience with condensates of dimension 10 since such high dimensions occur seldom in calculations. The good agreement of our previous calculation [14] with the results of BBK [13] may be considered as a support for applicability of the factorization procedure for dimension-10 condensates as well. In our opinion an estimated error of $\sim 50\%$ is a very conservative guess.

Let us now discuss the importance of the various contributions entering the expansion (25). As in [13, 14] the sum rule turns out to be dominated by the operators of highest dimension i.e., those of dimension 10. To be sure that this is a physical effect and not the onset of a breakdown of OPE one should reliably estimate the next term in the series, resulting in contributions of dimension 12, which is clearly very difficult. Contrary to the evaluation of the twist-3 matrix element $d^{(2)}$ where BPC did not contribute, local and bilocal corrections enter on equal footing in the present sum rule. The numerically important BPC turn out to be those involving the correlator with the axial current operator $\bar{q}\gamma_\alpha\gamma_5q$ and the correlator with the operator $\bar{q}\gamma_5\sigma_{\varphi\lambda}D_\alpha q$ for which an exact low energy theorem holds. The contributions from $\tilde{\delta}_\pi^3$ and $\bar{\delta}_\pi^3$ are numerically smaller and enter with opposite signs so that they partially cancel each other. Our final results rely on the dimension-10 condensates $\langle \bar{q}q \rangle^2$, m_0^2 , $\langle GG \rangle$, $\langle \bar{q}q \rangle^2$ and the bilocal correction $\langle \bar{q}q \rangle^2 \int d^4y \langle 0|TO_\sigma(y)(\bar{q}\gamma^\sigma\gamma_5q)(0)|0\rangle$. This situation is different from [13] where the contribution of the dimension-8 condensate $\langle \bar{q}q \rangle^2 m_0^2$ plays the crucial role. It is by no means trivial that a large number of different contributions merge together to give a result similar to that of BBK.

As we have mentioned already the introduction of an explicit gluonic component in the nucleon interpolating current resulted in a sum rule which to the lowest order is free of extra UV logarithms due to mixing, and therefore

the additional uncertainties discussed in [13, 20, 24] do not influence the final estimate. In the previous calculation of the twist-3 matrix element $d^{(2)}$ the mixing logarithm arose in the dimension-8 contribution which was numerically negligible in practice. So, as expected beforehand, the consideration of a non-perturbative gluonic component has lead to much less singular behaviour of the correlators and therefore to more stable numerical predictions.

In our previous calculation the values of the twist-3 matrix element where found to be $d^{(2)}(S) = -0.068 \pm 0.03$ and $d^{(2)}(NS) = 0.078 \pm 0.03$. Using these numbers we find from eqn. (5, 6) that both colour electric and colour magnetic fields in the rest system of the nucleon contribute at the same order of magnitude to the spin

$$\begin{aligned}\langle pS | gB^\sigma u^\dagger u | pS \rangle &= -(0.07 \pm 0.08)m_N^2 S^\sigma \\ \langle pS | gB^\sigma d^\dagger d | pS \rangle &= (0.188 \pm 0.08)m_N^2 S^\sigma \\ \langle pS | [(\bar{u}\vec{\gamma} u) \times g\vec{E}]^\sigma | pS \rangle &= (0.09 \pm 0.08)m_N^2 S^\sigma \\ \langle pS | [(\bar{d}\vec{\gamma} d) \times g\vec{E}]^\sigma | pS \rangle &= (0.21 \pm 0.08)m_N^2 S^\sigma .\end{aligned}\quad (31)$$

Obviously such a result shows that simple phenomenological models motivated as analogy to QED are misleading. In any such model one would expect the colour-magnetic term to dominate.

Finally we can analyze the higher-twist contributions to the integral over $g_1^p(x)$ and $g_1^{p-n}(x)$. If we estimate $a^{(2)}$ from experiment, $a^{(2)}(proton) = 0.022 \pm 0.002$ from EMC experiments [1] and $a^{(2)}(neutron) = 0.000 \pm 0.003$ from E142 [3] we obtain for the Bjorken sum rule

$$\int_0^1 dx g_1^{p-n}(x, Q^2) = \frac{1}{6}a^{(0)}(NS) + (0.003 \pm 0.008)\frac{\text{GeV}^2}{Q^2}, \quad (32)$$

where the twist-2 matrix element is defined as

$$2S^\sigma a^{(0)}(f) = \langle PS | \bar{q}_f(0) \gamma^\sigma \gamma^5 q_f(0) | PS \rangle. \quad (33)$$

In the case of the difference of proton and neutron structure function the flavor index refers to the non-singlet combination $u - d$. Using isospin symmetry this combination can be related to the nucleon β -decay constant $g_A/g_V = a^{(0)}(NS)$ leading to the celebrated Bjorken sum rule [5]. In case of the proton spin structure function we get

$$\int_0^1 dx g_1^p(x, Q^2) = \frac{1}{2}a^{(0)}(proton) - (0.015 \pm 0.007)\frac{\text{GeV}^2}{Q^2}. \quad (34)$$

Here $a^{(0)}(proton)$ contains non-singlet and singlet combinations. Using SU(3) symmetry it can be expressed by the F and D hyperon decay matrix elements. To compare with the leading twist matrix elements we take the Bjorken sum rule prediction for $a^{(0)}(NS)$ and for the Ellis-Jaffe leading twist element the measurement of SMC [2] taken at assumably asymptotic Q^2 .

$$\begin{aligned}\int_0^1 dx g_1^{p-n}(x, Q^2 \rightarrow \infty) &= \frac{1}{6} \frac{g_A}{g_V} = 0.2095 \pm 0.0005 \\ \int_0^1 dx g_1^p(x, Q^2 \sim 10\text{GeV}^2) &= 0.136 \pm 0.011 \pm 0.011. \quad (35)\end{aligned}$$

Thus we can conclude that the present analysis suggests that the higher-twist corrections to both sum rules are small for average Q^2 in the SMC and EMC range [1, 2].

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